g = relative recovery above the feed
H = vapor molal enthalpy
h = liquid molal enthalpy
K = vapor-liquid equilibrium composition ratio
L = liquid flow
l = liquid component flow
m = slope in a locally linearized equilibrium equation
p = pressure
Q = heat duty
S = component stripping factor
T = temperature
U = component recovery below the feed
u = relative recovery below the feed
V = vapor flow
v = vapor component flow
x = liquid mole fraction
y = vapor mole fraction
α = relative volatility

Subscripts

b = reboiler
c = condenser
e = estimated
h = heavy key
i = component index
l = light key
R = rectifying pinch
S = stripping pinch
w = feed stage
ϕ = stage above feed
ψ = stage below feed

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Convective Heat Transfer in Gas-Solid Suspension Flow Through Packed Beds

A correlation for the convective heat transfer between a bed of metallic oxides and a flowing gas-solid suspension was obtained experimentally. Four industrially important commercial catalysts with different physical, thermal, and transport properties were used as the bed materials. A wide range of air flow rates and solids to gas loading ratios were used. The correlating parameters, to estimate the convective Nusselt number, were found to be the Reynolds number, the Archimedes number, the solids loading ratio, and the shape factor of the packing materials. Impressive increases in heat transfer rates were observed with increased solids loading.

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SCOPE

In highly exo or endothermic gas phase solid catalyzed reaction systems, it is necessary to achieve enhanced convective heat transfer rates to obtain temperature control. One way of doing this is to entrain solid fines in the gas flowing through the bed. While a considerable amount of work has been done on gas-solid suspension flow through pipes and conduits (summarized by Depew and Kramer, 1973), very little information is available on the convective heat transfer in gas-solid suspension flow through packed beds.

This paper presents an experimental study to obtain a correlation for the convective heat transfer between the bed and the flowing gas-solid suspension. An experimental technique using microwave heating similar to that used by Balakrishnan and Pei (1974) in their studies on convective heat transfer from packed beds to gas flow alone was employed. The use of microwave power to heat a bed of metallic oxides results in the entire bed attaining a constant uniform temperature almost instantaneously. This eliminates thermal gradients within the bed and thereby the conduction mode.

Four industrially important commercial catalysts with different physical, thermal, and transport properties were used as the bed materials. Glass fines in air was used as the gas-solid suspension. Glass was chosen because it hardly absorbs any microwave power. A wide range of air flow rates and solids to gas loading ratios were used.

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CONCLUSIONS AND SIGNIFICANCE

From the experimental data obtained, a correlation to estimate the convective Nusselt number for heat transfer between a packed bed and a flowing gas-solid suspension has been proposed, namely

$$Nu_{fp} = 0.016[Ar_m]^{0.25}[Re_p]^{0.5}\phi_s^{3.76}[1 + \eta]^{0.68}$$

As can be seen, the correlating parameters were found to be the Reynolds number Re_p, the Archimedes number Ar_m, the solids loading ratio η, and the shape factor ϕ_s of the packing materials. Impressive increases in heat transfer rates were observed with increased solids loading. The dependence of Nusselt number on the Archimedes number and the Reynolds number indicates that the gravity force, the viscous force, and the inertia force are important factors in correlating the convective heat transfer rates in packed beds.

The effect of the Reynolds number seems to be to increase the Nusselt number, steeply at first, but then shows a tendency to level off. A possible explanation for this is that as Re_p increases, there will be increasing turbulence and therefore increasing convective heat transfer. However, once the flow is at a sufficiently high rate, any further increase in the flow rate will not increase the turbulence level significantly, and this accounts for the tendency to level off.

The Archimedes number is defined as the ratio of the gravitational force to the viscous force, and the convective Nusselt number increases as the one quarter power of the Archimedes number. When the gravitational force is large, the contact spots between the bed particles will also be large (for a randomly packed bed there are several contact spots). This means the surface area for convective heat transfer is relatively small for larger Ar_m and large for beds of small particles of low density. The dependence of Nusselt number can also be explained in terms of viscous forces. Low Archimedes numbers represent beds with particles of small diameters and low density with relatively viscous fluids flowing through them. On the other hand, high Archimedes numbers represent beds with large particles with lower viscosity flowing through them. Comparatively viscous fluids in beds with small particles will have less turbulence than less viscous fluids in beds with large particles.

As expected, impressive increases in convective heat transfer were observed with increased solids loading. Two mechanisms are possible for this. First of all, the fines may act as turbulence creators. Secondly, the fines may act as a heat carrier, physically absorbing the heat at the packing and releasing it to the bulk of the fluid.

This study clearly shows the introduction of inert fines in the gas stream to be a good method of promoting heat transfer from packed beds. It should be particularly useful in catalytic reactions with pronounced heat affects for maintaining temperature control.

There is a considerable amount of work on heat transfer from packed beds to flowing gases in the literature, and much of this work has been reviewed elsewhere (Barker, 1965; Balakrishnan and Pei, 1977) and will not be repeated here. Similarly, the large amount of work on heat transfer in gas-solid suspension flow through pipes and conduits has been critically reviewed by Depew and Kramer (1973). However, there is very little information available on the convective heat transfer from packed beds to flowing gas-solid suspension. Introducing inert fines in the gas stream flowing through a packed bed is suggested as a method of enhancing the convective heat transfer rates and therefore making the packed-bed system (such as a gas phase solid catalyzed reaction system) more amenable to temperature control. Royston (1971) experimentally studied the heat transfer from gas-solid suspensions to packed beds. The packed bed consisted of steel spheres, and the fines used were zircon, ilmenite, and ballotini. The experimental technique employed suggests the heat transfer coefficient obtained by Royston (1971) would include both the convective heat transfer as well as the particle to particle conduction in the bed. This is because of the existence of temperature gradients within the bed during the cooling cycle when the measurements were made. Furthermore, the correlation proposed by Royston

$$Nu_{ts} = \frac{Nu_{t(\text{with fines})} - Nu_{t(\text{without fines})}}{Nu_{t(\text{without fines})}} = 0.26\frac{c_{ps}}{c_{pf}}\eta \quad (1)$$

requires a knowledge of the heat transfer rates without fines, that is, $Nu_{t(\text{without fines})}$, to be able to estimate the heat transfer rates with the fines. This is a quantity that is generally not available easily. Moreover, since only one bed material, steel, was used, the effects of the properties of the bed material are not in evidence in the correlation.

The experimental technique to determine the convective heat transfer rates from a packed bed to a flowing gas laden with fine glass particles involved microwave heating. Ford and Pei (1967) and Bhattacharyya and Pei (1973) have shown that when a bed of metallic oxides is heated by microwaves, a constant uniform temperature is attained instantaneously. This results in the elimination of thermal gradients within the bed and thereby heat transfer by conduction. The use of the microwave technique in the present investigation is further facilitated by the fact that glass is transparent to microwave radiation. Therefore, the inclusion of glass fines as the suspension in the gas will not interfere with the measurement of the heat transfer rates from the bed to the flowing suspension. The function of the glass beads is thereby limited to promote turbulence, as is the case in most industrial situations.

THE EXPERIMENTAL SETUP

The complete experimental assembly is shown schematically in Figure 1. The apparatus consists essentially of a microwave power generator connected to a system of rectangular wave guides with the test section placed vertically through the wave guide. Microwave power, which is transmitted in a once-through process to heat up the solids in the test section, was generated by an Eimac (a division of Varian) PPS-215A power pack.

A circulator (Varian EW3-CL) was connected next to the generator to separate the forward and reflected power and

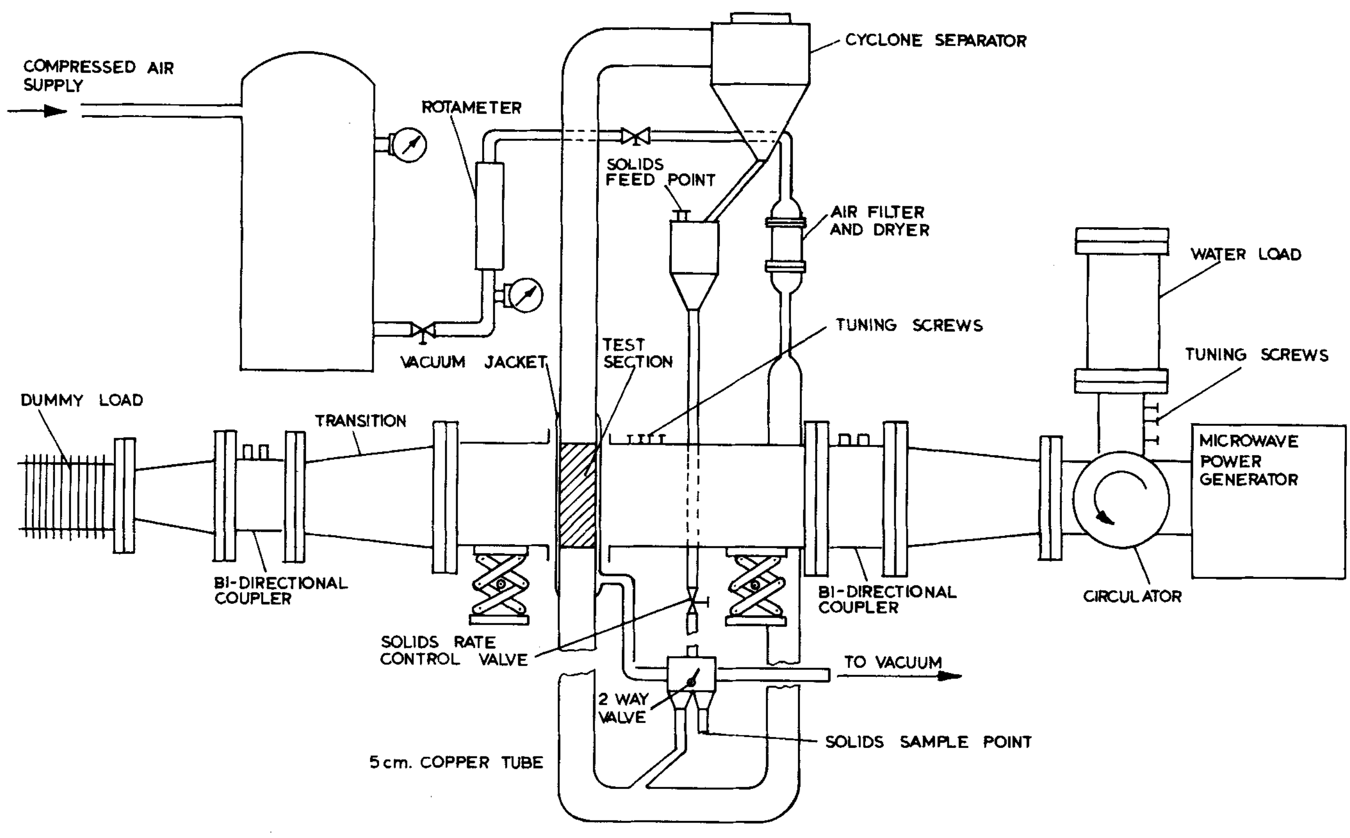

Fig. 1. Sketch of experimental setup.

divert the reflected beam to a water load for total absorption. This is done to avoid temperature fluctuation of the heated solids which will occur if the reflected beam is not diverted but allowed to couple with the generator output. The water load is primarily a device with circulating water which absorbs more than 99% of the incoming microwave power. The design details of the water load are available elsewhere (Balakrishnan, 1976). Design details of microwave power systems may be found in a number of handbooks and manuals (Baden Fuller, 1969; Barlow and Cullen, 1950; Hewlett Packard Co., 1963; Kelligian, 1965; Saad, 1971).

Compressed air supply at 3.1×10^5 N/m^2 gauge (45 lb/in.^{2}gauge) was used as the source of the pneumatic system with a stabilizing tank (0.45 m diam $\times$ 1.37 m high). The air then passed through a set of globe valves, a rotameter, and an air filter cum dryer. The stabilizing tank was provided with a safety/vent valve which could be set to any desired pressure [1.72×10^5 N/m^2 (25 lb/in.^{2}gauge) was maintained all through]. Air entered the 5.08 cm (2 in.) diameter test section through a 4.5 m (15 ft) high calming section of 5.08 cm ID copper tube. The test section was connected to the pneumatic system by short rubber hoses. The fines were entrained in the gas stream at the bottom of the pneumatic loop. The solids loading rate in the gas stream was controlled by a diaphragm valve. This rate could be monitored by collecting the fines at the sample point. A two way valve was used to direct all the solid flow to either the sample point or to the flowing gas stream to be entrained. The fines were recovered from the gas stream, after passing through the test section, in a cyclone separator and returned to the feed hopper.

The test section, shown in Figure 2, consisted of a vacuum jacketed Pyrex glass tubing; Pyrex is transparent to microwave radiation. Uniform exposure to the field was assured by placing the test section in the center of the wave guide. The outer jacket of the Pyrex glass tubing was connected to a vacuum pump (Welch Duo Seal Model 1405).

The glass tube within the wave guide was partially filled with solids to form the bed. Thermocouple wires to monitor the bed temperature were imbedded in some of the solids. Thermocouples were also used to measure the inlet and outlet gas temperatures. Copper constantan (Thermo Electric TG-30-T) thermocouples were used; the Teflon insulated singles were intertwined so that the field induced in one wire would cancel out that in the other. A filtering circuit, described by Bhattacharyya and Pei (1973), was also used to ground any A.C. field pickup.

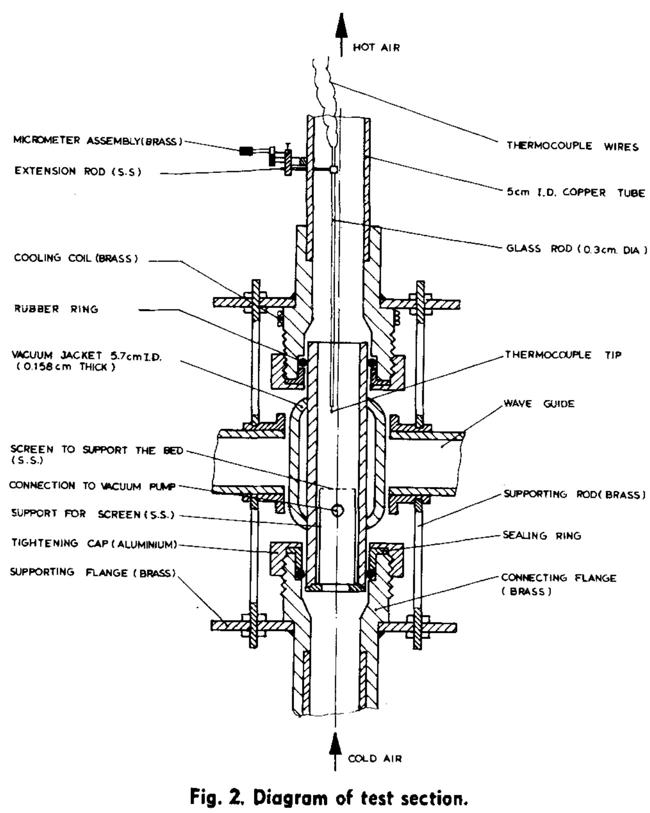

Fig. 2. Diagram of test section.

Table 1. Thermo-Physical Properties of Catalysts Used as Bed Materials

Catalyst	Shape	Size, cm	k_s, W/m · °K	c_{ps}, J/kg · °K	ρ_p, kg/m^3
Iron oxide	Spheres	0.635 (1/4 in.)	0.610 084	870.544	1 959.194
Nickel oxide	Spheres	1.27 (1/2 in.)	0.484 621	770.712	2 078.555
Vanadium pentoxide (type 1)	Cylinders ($L/D = 1$)	0.55 (7/32 in.)	0.452 414	1 737.522	1 345.55
Vanadium pentoxide (type 2)	Cylinders ($L/D = 1.5$)	0.55 (7/32 in.)	0.523 893	1 101.128	869.962

PROCEDURE

The packed bed in the test section consisted of commercial catalyst formulations, whose physical, thermal, and transport properties are listed in Table 1. To monitor the bed temperature, some of the solid particles were drilled and thermocouples inserted in them. The tagged particles are introduced into the test section and placed at randomly chosen locations. More solids are introduced into the test section to make up the entire bed.

The bed height is maintained at 4.8 cm, and the thermocouple lead wires are introduced as near to the wall as possible to minimize A.C. field pickup. These optimum conditions were arrived at by mapping the field distribution in the test section. The procedure for this field distribution study along with tests to verify the constant solid temperature assumption are described by Balakrishnan (1976).

A vacuum pump was used to maintain a pressure of 0.01 mm Hg in the test section jacket to ensure adiabatic conditions at the wall. Steady state measurements of inlet gas, outlet gas, and packing temperatures were carried out by recording their instantaneous values immediately after shutting off the microwave power. The microwave power was shut off to avoid field pick up in the recorders. The air flow rate was noted and the fines loading rate determined by measuring the volume of fines collected in a known amount of time at the sample point. The above procedure was repeated for different gas flow rates and different solids loading rates. As a precaution against the possibility of solids holdup in the bed, the lowest air flow rates used were at least three times the terminal velocities of the solid fines.

At steady state, the heat gained by the suspension on passing through the bed Q would be given by

$$Q = \dot{m}_f c_{pf}[(t_f)_{\text{out}} - (t_f)_{\text{in}}] + \dot{m}_s c_{ps}[(t_s)_{\text{out}} - (t_s)_{\text{in}}] \qquad (2)$$

However, $(t_f)_{\text{in}} = (t_s)_{\text{in}}$, and also it is likely that $(t_s)_{\text{out}} = (t_f)_{\text{out}} - \delta$, where δ approaches zero. Consequently, the heat transferred can be written as

$$Q = [\dot{m}_f c_{pf} + \dot{m}_s c_{ps}][(t_f)_{\text{out}} - (t_f)_{\text{in}}] \qquad (3)$$

Therefore, a complete heat balance on the bed yields

$$h_{fp} = \frac{\dot{m}_f c_{pf} + \dot{m}_s c_{ps}}{A_{pe}} \cdot \ln \frac{[t_p - (t_f)_{\text{in}}]}{[t_p - (t_f)_{\text{out}}]} \qquad (4)$$

the convective heat transfer coefficient.

Figures 3 to 6 show the experimental data as a plot of fluid particle heat transfer coefficient (Nusselt number) vs. particle Reynolds number for different loading rates. Each figure refers to one particular bed material with three sizes of glass fines. These, apart from differing in size also, vary in density (Table 2). Therefore, even though adjusted for the same volumetric rate of entrainment, the mass solids loading rates vary.

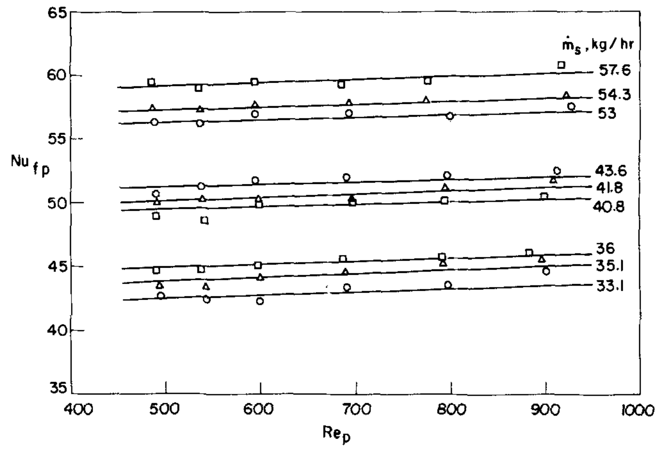

Fig. 3. Experimental data, 0.635 cm iron dioxide spheres. △. 100μ glass fines, ⬡. 150μ glass fines, ○. 250μ glass fines.

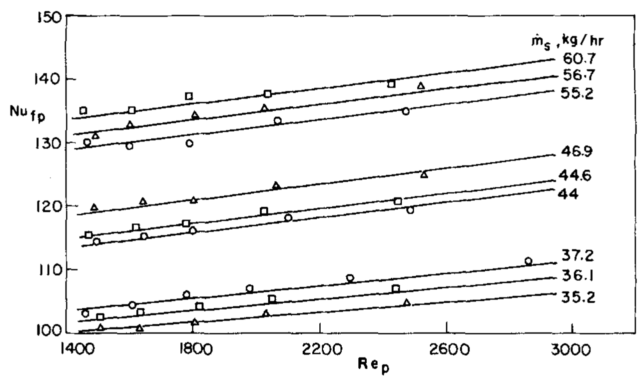

Fig. 4. Experimental data, 1.27 cm nickel oxide spheres. △. 100μ glass fines, ⬡. 150μ glass fines, ○. 250μ glass fines.

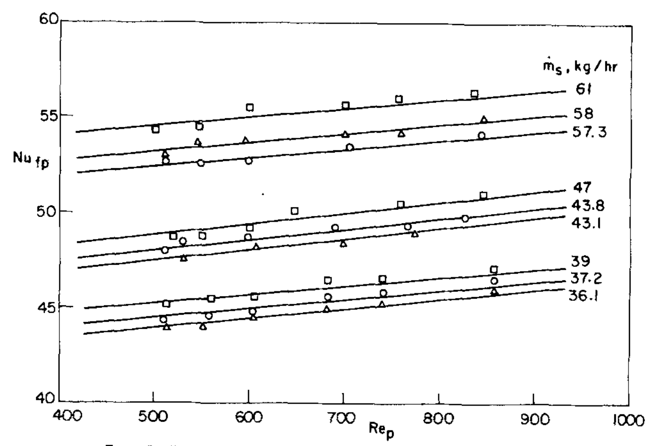

Fig. 5. Experimental data, 0.55 cm V_2O_5 (type 1). △. 100μ glass fines, ⬡. 150μ glass fines, ○. 250μ glass fines.

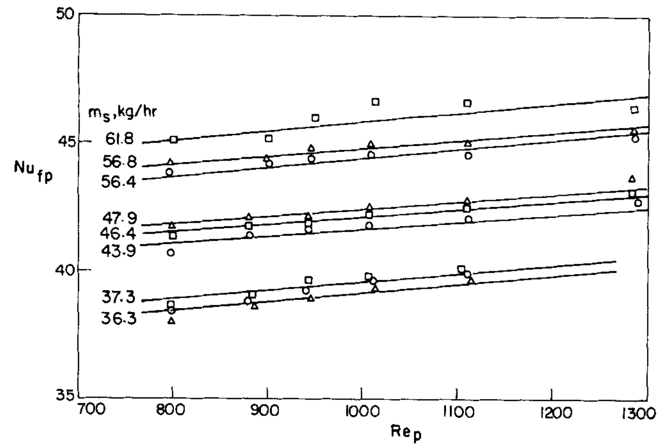

Fig. 6. Experimental data, 0.55 cm V_2O_5 (type 2).
△. 100μ glass fines, ○. 150μ glass fines, ○. 250μ glass fines.

TABLE 2. PHYSICAL PROPERTIES OF FINES

Material	Characteristic size, μ	Size range, μ	Bulk density kg/m³	lb/ft³
Blast-O-lite	100	88-149	1 413.2	88.2
	150	125-177	1 434.5	89.5
Industrial glass beads*	250	210-297	1 443.5	90.0

* Generally used for sand blasting.

From the plots of Figures 3 to 6, it is apparent that for a particular packing material, an increase in solids loading rates results in an increase in the bed to fluid heat transfer rate, namely, Nu_{fp} (or h_{fp}). Another observation which is readily discernable is that for different bed materials, the absolute values of h_{fp} show a wide variation even for approximately identical solids loading rates and particulate Reynolds numbers. The approximately straight lines drawn through the data points in the figures correspond to the correlation developed in the next section (that is, a least-square fit of all the data points collected).

DATA CORRELATION

From Figures 3 to 6, it is apparent that the rate of heat transfer depends on the gas flow rate, the loading rate, and the properties of the bed material [each bed yields a different heat transfer rate for the same (or similar) loading rate]. In developing a general correlation for the data, the following two criteria were observed: the expression should be dimensionless in form, and it should be based on easily determined parameters of the system.

In earlier studies on convective (fluid particle) heat transfer in gas fluidized beds (summarized by Davidson and Harrison, 1971; Gutfinger and Abauf, 1974; Zabrodsky, 1966), the parameters frequently used to correlate the data were the Archimedes number and the Reynolds number. Bhattacharyya and Pei (1975), who obtained convective heat transfer rates in packed beds subject to gas flow alone, also correlated their data in terms of these two parameters:

$$jh_{fp} = 0.018\left[\frac{Ar_m}{Re_p^2}\right]^{0.25} \quad (5)$$

However, the Colburn J factor jh_{fp} is not a very satisfactory parameter to correlate convective heat transfer data. This is because it has included in it a velocity term. Therefore, correlations such as Equation (5) have velocity on both sides of the equation. It is preferable instead to use the conventional Nusselt number in correlating heat transfer rates. Furthermore, for the convective heat transfer from a packed bed to flowing gas-solid suspensions, the loading ratio has to be considered. Thus, an attempt is made here to correlate the data in the form

$$\frac{Nu_{fp}}{Ar_m^{0.25} Re_p^{0.5}} = A(1+\eta)^c \quad (6)$$

where η is the loading ratio of the solid fines, and A and c are constants which have to be determined (using a least-squares fit) from experimental data. The correlation thus arrived at for beds with spherical particles is

$$Nu_{fp} = 0.016[Ar_m]^{0.25}[Re_p]^{0.5}[1+\eta]^{0.68} \quad (7)$$

Balakrishnan and Pei (1974), in an earlier study, modified Bhattacharyya and Pei's (1975) correlation [Equation (5)] to include the effect of using beds with non-spherical particles. They suggested that the dependence of the convective heat transfer on the shape factor is

$$Nu_{fp(\text{any shape})} = Nu_{fp(\text{sphere})} \cdot \phi_s^{3.76} \quad (8)$$

The reference sphere in Equation (8) is defined as a sphere having the same volume as the arbitrarily shaped particle. If we adopt the same approach, Equations (7) and (8) may be combined to yield

$$Nu_{fp} = 0.016[Ar_m]^{0.25}[Re_p]^{0.5}\phi_s^{3.76}[1+\eta]^{0.68} \quad (9)$$

The shape factor ϕ_s, also called the sphericity factor, is defined as (Leva, 1959)

$$\text{Shape factor } \phi_s = \frac{A_s}{A} \quad (10)$$

The spherical particle of surface area A_s has the same volume as the arbitrarily shaped particle of surface area A. Since a spherical particle is a body that will provide a given mass with the least surface area, values calculated by Equation (10) will always be less than unity.

The reason behind using $[1+\eta]$ in the correlation [Equation (9)] is that at $\eta \to 0$, the equation should yield the heat transfer rate for a bed subject to gas flow alone (that is, zero solids loading).

As can be seen from Equation (9), the dimensionless convective heat transfer coefficient (Nusselt number) depends on the following four parameters: the loading ratio, η; the ratio of inertial forces to the viscous forces, Re_p; the ratio of gravitational forces to the viscous forces, Ar_m; and the shape factor of the materials which constitute the bed.

The standard error of estimate of the correlation is 8%. The experimental range of the parameters are

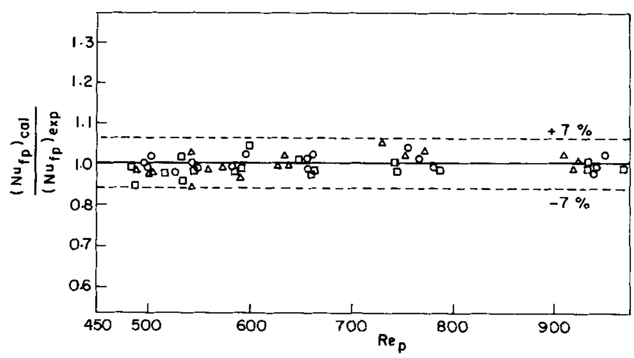

Fig. 7. Performance of correlation, 0.635 cm iron oxide spheres.
△. 100μ glass fines, ○. 150μ glass fines, ○. 250μ glass fines.

$$481 < Re_p < 3\,759$$
$$5\,400\,000 < Ar_m < 43\,000\,000$$
$$0.9 < \eta < 5.4$$
$$0.85 < \phi_s < 1.0$$

The dependence of Nusselt number on Ar_m and Re_p clearly indicates that the gravity force, the viscous force, and the inertia force are important factors in correlating the convective heat transfer rates in packed beds. To examine the performance of the correlation, the ratio of predicted and observed values of Nu_{fp} was plotted as a function of Reynolds number. Figure 7, representing such a plot for 0.635 cm iron oxide spheres, illustrates the fact that there is no residual dependence of Nu_{fp} on Re_p. A separate figure was used for each bed material in view of the very large number of data points. The maximum deviation obtained is 11%.

It was found that the fit of the data to the correlation is much better for spheres than it is for cylinders. The maximum deviation for the spheres is less than 7%, but for the cylinders it is 9 and 11% for the two samples used. Since the range of ϕ_s in the present investigation [0.85 to 1] is very narrow, more work is needed to either confirm or modify the correlation with respect to the shape factor. Another reason for the greater scatter of the data for cylinders than for spheres could be due to the fact that the L/D ratio specified by the manufacturers of the cylindrical catalyst samples is true only on the average. This would undoubtedly cause an uncertainty in the estimation of the shape factor ϕ_s.

DISCUSSION

In view of the fact that the convective Nusselt number Nu_{fp} depends on four different parameters, it may be useful to plot Nu_{fp} vs. each of these parameters, keeping the others constant. In this manner, a qualitative picture of the dependence of Nu_{fp} on each of these parameters may be obtained.

Figure 8 shows a plot of Nu_{fp} vs. Re_p for various values of the loading ratio η. The Archimedes number (modified) Ar_m was kept constant at 59 000 000. The figure indicates that Nu_{fp} tends to increase very rapidly at first and then has a tendency to level off. At $Re_p \to 0$, Nu_{fp} is also zero. This is as it should be for at zero Re_p, that is, at no flow, there can be no convective effects. As Re_p increases, there will be increasing turbulence and therefore increasing convective heat transfer. A plausible explanation for the tendency of Nu_{fp} to level off at higher Reynolds number is that once the flow is at a sufficiently high rate that there is a considerable amount of turbulence, any further increase in flow rate will not increase the turbulence level significantly. In other words, the turbulence level and therefore the convective heat transfer reach an asymptotic level with Re_p. However, the introduction of entrained fines changes the picture quite a bit, as seen in Figure 8. The fines seem to enhance the heat transfer rates appreciably. This enhancement could be attributed to two different mechanisms. First of all, the fines may act as turbulence creators. As more fines are entrained (increasing the loading ratio η), the turbulence level increases and greater heat transfer takes place, even at the same Reynolds number Re_p. Secondly, the fines may act as a heat transfer carrier. They might absorb heat at the surfaces of the packing, physically move into the bulk of the flowing gas, and then release it there. However, the direct heat transfer from the bed material to the fines will be very small owing to the limited time and area of contact between the fines and the bed material. It is therefore believed that the mechanism of turbulence promotion is more important.

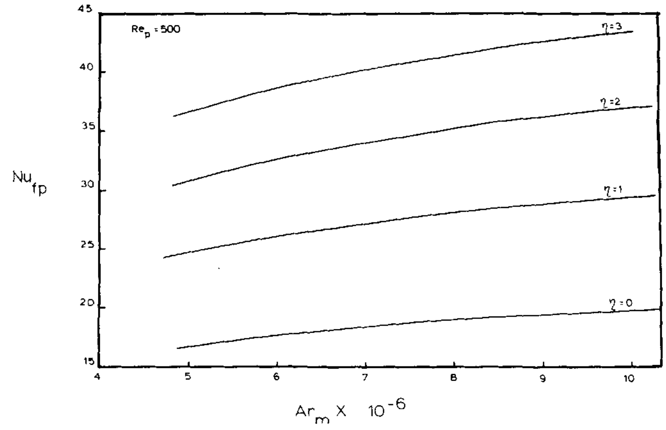

Fig. 9. Dependence of Nu_{fp} on Ar_m.

Figure 9 shows the dependence of Nu_{fp} on the modified Archimedes number Ar_m. The Reynolds number was maintained constant at 500 and the shape factor at 1. As per the correlation [Equation (8)], Nu_{fp} increases as the fourth root of Ar_m. To understand the physical significance of Figure 9, consider the definition of the modified Archimedes number

$$Ar_m = \frac{D_p^3 g \rho_f (\rho_p - \rho_f)(1-\epsilon)^2}{\mu_f^2} = \frac{\text{gravitational force}}{\text{viscous force}} \tag{11}$$

When the gravitational force is large, the contact spots between the bed particles will also be large (for a randomly packed bed there are several contact spots). This means the surface area for convective heat transfer is relatively small for large Ar_m. Similarly, for beds with particles of small diameter and low density, the contact spot dimensions will be small, and therefore the area for convection will be relatively large.

The dependence of Nu_{fp} on Ar_m can also be explained in terms of the viscous forces. In Figure 9, the lower end of the horizontal axis represents beds with particles of small diameters and low density with relatively viscous fluids flowing through them. On the other hand, the far end of the horizontal axis refers to beds with large particles of high density with fluids of lower viscosity. Comparatively viscous fluids in beds with small particles will have less turbulence (lower Ar_m values) than less viscous fluids in beds with large particles (high Ar_m values). This also explains, in a qualitative manner, the reason for increasing Nu_{fp} values with increasing Ar_m.

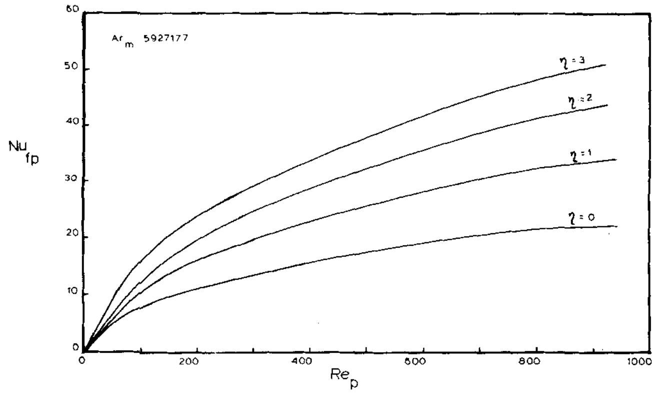

Fig. 8. Dependence of Nu_{fp} on Re_p.

CONCLUSIONS

Based on the results of this experimental study, the following conclusions can be drawn:

1. The convective heat transfer from a packed bed to a flowing gas-solid suspension was measured using microwave heating.
2. Analysis of the data led to a correlation which may be used to estimate the convective Nusselt number.
3. The important correlating parameters were found to be the particle Reynolds number, the Archimedes number, and the loading ratio.

NOTATION

A = surface area of arbitrarily shaped particle
$A_{p \cdot e}$ = surface area of the particles in the bed, m^2
A_s = surface area of spherical particle
A_t = cross-sectional area of test section, m^2
Ar_m = Archimedes number, $D_p{}^3 g \rho_f(\rho_p - \rho_f)\ (1 - \epsilon)^2/\mu_f{}^2$, dimensionless
c_{pf} = specific heat of fluid, J/kg·°K
c_{ps} = specific heat of fines, J/kg·°K
D_p = diameter of particles in bed, m
G_f = fluid mass velocity, kg/m^2·s
G_s = fines mass velocity, kg/m^2·s
h_{fp} = convective heat transfer coefficient, J/m^2·s·°K
jh_{fp} = Colburn-J factor for convective heat transfer, $(h_{fp}/c_{pf}G_f)\,Pr^{2/3}$, dimensionless
k_f = fluid thermal conductivity, J/m·s·°K
$\dot{m}_f$ = fluid mass flow rate, kg/s
$\dot{m}_s$ = solid (fines) flow rate, kg/s
Nu_{fp} = convective Nusselt number, $h_{fp} \cdot D_p/k_f$, (dimensionless)
Nu_t = total Nusselt number, Nu_{ts}, increase in Nusselt number due to fines
Pr_f = Prandtl number, $c_{pf} \cdot \mu_f/k_f$, dimensionless
Re_p = Reynolds number, $D_p \cdot u_f \cdot \rho_f/\mu_f$, dimensionless
t = temperature, °K, t_p of packing, t_f of fluid
u_f = fluid velocity, m/s

Greek Letters

ϵ = bed porosity, dimensionless
η = loading ratio $(\dot{m}_s/\dot{m}_f)$ dimensionless
μ_f = fluid viscosity, N·s/m^2
ρ_p = bed particle density, kg/m^3
ρ_f = fluid density, kg/m^3
ϕ_s = shape factor, dimensionless

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Reliability of Optimization Procedures for Obtaining Global Optimum

The importance of the starting point, the size of initial search region, and the search region reduction rate is examined with respect to the reliability of different direct search optimization procedures in being able to furnish the global optimum for nonunimodal systems. Although, in general, the reliability of an optimization procedure is problem dependent, it is nevertheless clear that reliability cannot be increased simply by selecting larger search regions or by reducing the rate of contraction of the search region. A more efficient means of increasing reliability is to embody a pseudo one-dimensional search in the optimization procedure to enable the search to leave a local optimum and proceed to a better optimum.

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SCOPE

The ease of programming and the ease with which inequality constraints can be handled make direct search optimization procedures attractive from user's point of view. Luus and Jaakola (1973) presented a direct search method (LJ method) based on random sampling and search region contraction. The method is easy to program and is also computationally efficient for solving general nonlinear programming problems.